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Quantum interference due to Larmor precession in mesoscopic loop structures

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Abstract. A simple theory of the quantum interference in a loop structure due to Larmor precession of the electron spin is presented in this paper. A 'spin ballistic regime' is assumed, where the phase-relaxation length for the spin part of the wave function ($L_\varphi^{(s)}$) is much greater than the relaxation length for the 'orbital part' ($L_\varphi^{(e)}$). In the presence of an additional periodic magnetic field, the spin part of the electron's wave function acquires a phase shift due to additional spin precession about that field. If also the structure length L is chosen such that $L_\varphi^{(s)} > L > L_\varphi^{(e)}$, it is possible to 'wash out' the quantum interference related to the phase coherence of the 'orbital part' of the wave function, retaining at the same time that related to the phase coherence of the spin part and, hence, to reveal the corresponding conductance oscillations. It is also shown that strong modulation of the interference pattern could be achieved in this case.

1. Introduction

Most transport phenomena in condensed matter are described by the Boltzmann kinetic equation and are classical in nature. However, there are many quantum effects in solid-state physics which involve the interference of electron waves; among them is the Aharonov–Bohm (AB) effect [1]. Oscillatory magnetoresistance due to the Aharonov–Bohm effect has been observed in small metallic rings [2] and in the microstructure consisting of a semiconductor (GaAs) loop embedded in GaAlAs [3]. A number of papers were concerned with the origin of these oscillations (h/e as well as $h/2e$); these works treated the charge-carrier transport as diffusive, however.

In their well-known paper [4], Datta and Bandyopadhyay considered ballistic transport, and they have shown that it is possible (in principle) to approach even 100% conductance modulation in a magnetic field. The conditions which the microstructure has to obey are however very strict. In particular, there should be a 'single-mode' regime, which is certainly difficult to achieve. It should be noted that, so far, researchers have mainly considered the Hamiltonian $\hat{H} = (\boldsymbol{p} - (e/c)\boldsymbol{A})^2/2m^* + U(y)$, where $U(y)$ is the energy corresponding to the transverse motion, and almost nobody has taken into account the spin part $\mu\hat{\sigma}\boldsymbol{B}$ of the Hamiltonian (μ is the Bohr magneton, $\hat{\sigma}$ is the electron spin operator, $\boldsymbol{B}$ is the magnetic field). The main reason for neglecting this term is that the Bohr magneton is very small and, as a result, the energy of the electron charge interaction with the external field is much greater than that of the spin interaction. However, generally speaking, the spin part of the electron wave function can also acquire a phase shift in the course of the electron evolution in a magnetic field and hence cause the conduction of the microstructure to oscillate. The aim of this paper

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is to present a simple theory of quantum interference in a semiconductor mesoscopic structure which is due to Larmor precession of the spin in a spatially periodic magnetic field under conditions of 'spin ballistic transport'.

2. The model and necessary preliminaries

Let us consider a generic microstructure with two end regions ($x < 0$ and $x > L$) and a middle region $0 \leqslant x \leqslant L$, consisting of two channels (see figure 1), similar to the one considered in [4]. The main difference, however, is that here the external magnetic field $\boldsymbol{B}_0$ is in the plane of the microstructure and, in addition, on the upper surface of one of the channels there is a regular array of micromagnets similar to that discussed by von Klitzing and co-workers [5]. It should be noted that, in fact, the only thing which is really needed is that the magnetic fields have to be different in the two arms of the loop. The periodic field is not obligatory, and this choice is motivated by the current interest in the study of electron motion in inhomogeneous magnetic fields on the nanometre scale [6].

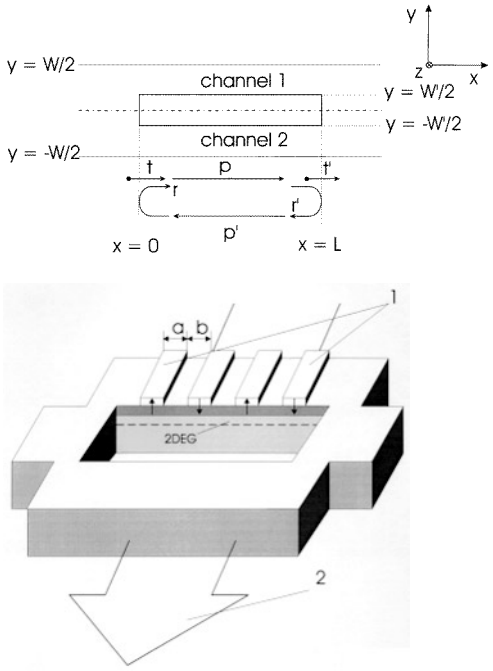

Figure 1. A sketch of a two-channel semiconductor microstructure with a magnetic grating on the top of one of the channels. t, t', r, r' indicate the transmission and reflection matrices at the two junctions $x < 0, x > L$; P, P' stand for the propagation matrices in the middle region ($0 < x < L$). 1: micromagnets (a one-dimensional ferromagnetic grating); 2: the external magnetic field $\boldsymbol{B}_0$.

Suppose these micromagnets create within this channel an additional periodic magnetic field $\boldsymbol{B}_1$ of the form

$$\boldsymbol{B}_1(0, 0, B_1) = B_1(x) = \begin{cases} B_1 & \text{if } 2n(a+b) \leqslant x \leqslant 2n(a+b)+a \\ 0 & \text{if } n(a+b)+a < x < (n+1)(a+b) \\ -B_1 & \text{if } (2n+1)(a+b) \leqslant x \leqslant (2n+1)(a+b)+a \end{cases} \tag{1}$$

where $n = 0, 1, 2, \ldots$. Here a is the width of the magnetic strip and b is the spacing, so $a + b$ is the half-period of the magnetic grating. This means that the magnetic field $\boldsymbol{B}$ which affects the electron in the first arm of the structure is equal to $\boldsymbol{B} = \boldsymbol{B}_0$, while in the other arm it is equal to $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1$. Suppose that the Hamiltonian of the electron is $H = H_0 + H_1$, where

$$H_0 = (1/2m^*)(\boldsymbol{p} - (e/c)\boldsymbol{A})^2 + U(\boldsymbol{r}) \qquad H_1 = -\mu_B \hat{\sigma}\boldsymbol{B}. \tag{2}$$

Here m^* is the electron effective mass, $\boldsymbol{A}$ is the vector potential corresponding to the magnetic field $\boldsymbol{B}$, μ_B and $\hat{\sigma}$ are the Bohr magneton and the spin operator respectively. We also assume that $U(\boldsymbol{r})$ describes conduction band bending due to the space charge and discontinuities of any band. Since H_0 does not depend on the spin, the wave function is a direct product: $\Psi(\boldsymbol{r}, s) = \varphi(\boldsymbol{r}) \otimes \chi(s)$. For convenience, we will refer to $\varphi(\boldsymbol{r})$ as the 'orbital part' of the total wave function, keeping in mind that it corresponds to H_0 describing the charge–field interaction, and we will refer to $\chi(s)$ as the spin part of the wave function related to H_1, the spin part of the Hamiltonian H in (2).

Let us now introduce the phase-relaxation length $L_\varphi^{(s)}$ for the spin part of the wave function, in just the same way as the one usually introduced for the 'orbital part', $L_\varphi^{(e)}$. Our main hypothesis is that the phase-relaxation length $L_\varphi^{(s)}$ is much greater than $L_\varphi^{(e)}$. The reason for this is quite simple: the electron spin–phonon interaction is much smaller than the electron charge–phonon interaction. However, it is necessary to make some additional remarks. As is known, as a rule rigid scatterers such as impurities and other defects of crystalline structure do not contribute to the phase relaxation; only dynamical scatterers like phonons do. But impurity scattering can also be phase randomizing if the impurity has an internal degree of freedom with the result that it can change its state. For example, if magnetic impurities have an internal spin that fluctuates with time, the collisions with such impurities cause phase relaxation. So, we suppose there are no such impurities here.

Now if we suppose the microstructure length L to be chosen such that $L_\varphi^{(s)} > L > L_\varphi^{(e)}$, it is possible to 'wash out' the quantum interference related to the phase coherence of the 'orbital part' of the wave function, retaining at the same time that related to the phase coherence of the spin part, and, hence, to reveal the corresponding conductance oscillations.

Let us add some more comments concerning the phase-relaxation length $L_\varphi^{(s)}$ which is connected to the corresponding phase-relaxation time $\tau_\varphi^{(s)}$. It is clear that the electron motion over a time $\tau_\varphi^{(s)}$ being 'spin ballistic' is not ballistic in the usual sense of the word. It means here that if the momentum-relaxation time $\tau_m \ll \tau_\varphi^{(s)}$ (which, as we shall see later, is actually the case), after a time interval $\sim\tau_m$ the electron velocity becomes randomized, so the electron trajectory over a period of time $\tau_\varphi^{(s)}$ can be represented as the sum of a number ($\sim\tau_\varphi^{(s)}/\tau_m$) of short trajectories each of the length $\sim v_F\tau_m$ (v_F is the Fermi velocity), just as is usually done for the common relaxation time $\tau_\varphi^{(e)}$ (see, for example, [7]). Since the individual trajectories are directed at random (the directions are determined by the angle α), the root mean square distance travelled by electrons in a particular direction is obtained from the squares of their lengths:

$$L_\varphi^{(s)2} = \frac{\tau_\varphi^{(s)}}{\tau_m}(v_F\tau_m)^2\langle\cos^2\alpha\rangle \qquad \langle\cos^2\alpha\rangle = \int_{-\pi}^{\pi}\frac{\mathrm{d}\alpha}{2\pi}\,\cos\alpha = 1/2$$

and hence

$$L_\varphi^{(s)} = v_F\sqrt{\tau_m\tau_\varphi^{(s)}/2}.$$

It should be noted that the last relation is also valid for $L_\varphi^{(e)}$ with $\tau_\varphi^{(e)}$ substituted for $\tau_\varphi^{(s)}$, if $\tau_m < \tau_\varphi^{(e)}$ which is often the case.

In order to estimate the phase-relaxation time $\tau_{\varphi}^{(s)}$, consider a simple model. Let us take a two-state quantum system (which we shall refer to as subsystem $\mathcal{A}$) with excitation energy ε interacting with a phonon bath, and identify the two states with 'spin-up' ($|\uparrow\rangle$) and 'spin-down' ($|\downarrow\rangle$) states of a spin in an external magnetic field. For simplicity, we suppose the interaction of the subsystem $\mathcal{A}$ with the phonons to be resonant; this means that only those modes of the phonon bath whose energy is equal to ε interact with the two-level subsystem. Other modes are taken into account indirectly by choosing all mean values of phonon bath parameters to be equal to their statistical average values at a given temperature T. As a result, for the model of the phonon bath we can take a great number ($N \gg 1$) of identical non-interacting subsystems $\mathcal{B}_n$ with excitation energy ε.

Thus, the Hamiltonian of the entire system (subsystem $\mathcal{A}$ + phonon bath) is

$$H = \varepsilon\left(a^{\dagger}a + \sum_{n=1}^{N} b_n^{\dagger}b_n\right) \tag{3}$$

where $a^{\dagger}, a$ are the Fermi creation and annihilation operators related to the excitations of subsystem $\mathcal{A}$, while $b_n^{\dagger}$, b_n are Bose creation and annihilation operators related to the excitations of the of the nth subsystem of the phonon bath.

So, for the subsystems of the equidistant spectra, we have

$$\langle b_n^{\dagger}b_n\rangle = \mathrm{Sp}(\rho_n b_n^{\dagger}b_n) = (\exp(\beta\varepsilon) - 1)^{-1} \qquad \beta = 1/k_B T$$

where ρ_n is the statistical operator for the $\mathcal{B}$ subsystems, k_B is the Boltzmann constant, T is the temperature, $\mathrm{Sp}(\cdots)$ is the trace operator.

The interaction of two-level systems with phonons can be described by the term $\mathcal{H}_{int}(t)$:

$$\mathcal{H}_{int}(t) = \sum_{n=1}^{N}[\theta(t - \tau(n-1)) - \theta(t - \tau n)]H_n \tag{4}$$

where

$$\theta(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t \leqslant 0 \end{cases}$$

$$H_n = \varepsilon_{int}(a^{\dagger}b_n + b_n^{\dagger}a)$$

and ε_{int} is the interaction energy.

The physical meaning of (4) is that the subsystem $\mathcal{A}$ interacts at each time during the interval τ with those subsystems $\mathcal{B}_n$ which did not interact with $\mathcal{A}$ during the previous time interval, or, in other words, τ is the 'electron–phonon collision time'. As was mentioned above, the rigid scatterers do not contribute to the phase relaxation; only the dynamical—that is, time-dependent—scatterers, such as phonons, do; since we are interested in the estimation of the phase-relaxation time, the Hamiltonian (4) is explicitly time dependent.

We introduce now two probabilities $p_1(t)$ and $p_2(t)$ for the subsystem $\mathcal{A}$ to be, at a time t, in an excited state and an unexcited one, respectively. It is well known that the steady state for the two-level system corresponds to $p_1 = p_2 = 1/2$. As can be shown (see appendix 1), for the model described above, the time t which is needed for the subsystem $\mathcal{A}$ to achieve the state with $p_1 = p_2 = 1/2$ is equal to

$$t = (\hbar^2/\tau\varepsilon_{int}^2)\ln 2\tanh(\beta\varepsilon/2). \tag{5}$$

Since the two levels of the subsystem $\mathcal{A}$ correspond to the 'spin-up' and 'spin-down' states, the steady state corresponds to the redistribution of the initially non-equilibrium spin distribution due to spin-flip transitions and, hence, to the total destruction of spin coherence.

Thus, time t is the spin-relaxation time, which can be identified with $\tau_\varphi^{(s)}$ because it relates to the phase-coherence destruction caused by inelastic scattering by the spin flips.

In order to estimate $t \sim \tau_\varphi^{(s)}$ by means of (5), we should estimate first the interaction energy ε_{int}. One should note that spin flips are possible only when there is spin–orbit interaction [8]. Thus, considering the electron scattering by acoustic phonons, in order to describe the spin flips one should take into account the spin–orbit interaction. If we take it to be $\varepsilon_{int} = (e\overline{h}/2mc)^2 a_0^{-3}$, where a_0 is of the order of the Bohr radius, τ is about 10^{-15} s (remember that the physical meaning of τ is the 'electron–phonon collision time' which could be estimated as follows: the shortest momentum-relaxation time is for metals and of the order of 10^{-15} s, but the 'collision time' cannot be greater than the momentum-relaxation time), the magnetic field B is of the order of 1 T, while the temperature is about 5 K, then for the time $t \sim \tau_\varphi^{(s)}$ we have the result $\sim 2.2 \times 10^{-10}$ s.

The values of the spin-relaxation time measured for the complex semiconductors of the third and fifth groups ($A^{III}B^{V}$) range widely from 10^{-12} to 10^{-7} s [9], and hence we can conclude that our estimate for $\tau_\varphi^{(s)}$ is quite reasonable. Anyway, we can take it as certain that $\tau_\varphi^{(s)} \gg \tau_\varphi^{(e)}$. Indeed, experiments show [10] that at 5 K the phase-relaxation time $\tau_\varphi^{(e)}$ is about 1.6×10^{-12} s and, as a result, $L_\varphi^{(s)} \gg L_\varphi^{(e)}$; hence, the structure length L can be chosen to be such that $L_\varphi^{(s)} > L > L_\varphi^{(e)}$.

3. Calculation of the transmission coefficient

The current I through the structure considered in the previous section, for the small applied potential V, can be written as [4, 7]

$$I = \frac{2e}{h} \int \mathrm{d}E \int (w_z \, \mathrm{d}k_z/2\pi) \, [f(E) - f(E + eV)] \sum_{n',n''} |T_{n',n''}|^2. \tag{6}$$

Here w_z is the width of the structure in the z-direction, $T_{n',n''}$ is the coefficient of transmission from the state n' in the left-hand end to the state n'' in the right-hand end, E and k_z are the energy and the transverse wave vector of the electrons as they enter from the left-hand end.

Let us suppose for a moment that the structure length is smaller than the phase-relaxation length; then the charge transport is said to be coherent and one calculates the transmission coefficient starting from the Schrödinger equation with the Hamiltonian (2). It is well known [7] that a coherent conductor can be characterized at each energy by an S-matrix that relates the outgoing wave amplitudes to the incoming wave amplitudes in the different leads. Thus, the transmission coefficient $T_{n',n''}$ can be obtained by taking the squared magnitude of the corresponding element of the S-matrix. Taking into account the relation $\Psi(\boldsymbol{r}, s) = \varphi(\boldsymbol{r}) \otimes \chi(s)$ and using the property of the direct product $(A \otimes B)(C \otimes D) = AC \otimes BD$, one can demonstrate that $T_{n',n''} = T_{k',k''} \otimes T_{\sigma'\sigma''}$ where subscripts k', k'' relate to the states of H_0, while subscripts σ', σ'' relate to the states of H_1. Now take into account the fact that the structure length L is greater than $L_\varphi^{(e)}$. Then, dividing the structure into sections of length smaller than $L_\varphi^{(e)}$, one can combine the successive scatters [7] and treat the transport through the states k', k'' as incoherent, while the transport through the states σ', σ'' is coherent, because $L < L_\varphi^{(s)}$. As a result, we have $T_{n',n''} = \langle T \rangle T_{\sigma'\sigma''}$, where $\langle T \rangle$ is the averaged transmission coefficient which does not depend on the phase relation between the states of H_0 in the left-hand end and in the right-hand end of the structure. So, in accordance with the assumptions above, there are two states ('spin up' and 'spin down') to consider in the end regions, while in the middle region there are four states corresponding to the channels 1 and 2. Dropping the subscripts σ', σ'',

one can write down the following expression for the transmission coefficient T [11]:

$$T = t' \left[I - Pr P' r' \right]^{-1} Pt. \tag{7}$$

Here I is the unit matrix, t is a 4×1 matrix describing the transmission from the left-hand end into the two channels, while t' is a 1×4 matrix describing the transmission from the channels into the right-hand end. Similarly, r and r' are 4×4 matrices describing the reflections at the two junctions of the channels back into the channels. Matrices P and P' describe forward and reverse propagation of the electron wave through the channels 1 and 2, respectively. In order to construct the matrices r and r', let us suppose $i, j = 1, \ldots, 4$ each stand for one of four states: 'spin up' or 'spin down' in the channels 1 or 2. Then the r_{ii} stand for the scattering from a state of definite spin ('spin up' or 'spin down') to the same state (in other words, for 'self-scattering') in the channels 1 or 2 at the first junction; say, r_{11} means $|\uparrow\rangle \rightarrow |\uparrow\rangle$ scattering in the first channel, r_{33} means $|\uparrow\rangle \rightarrow |\uparrow\rangle$ scattering in the second channel and so on. The same is true for r'_{ii} but at the second junction, while r_{ij}, r'_{ij} stand for the scattering from the 'spin-up' state to the 'spin-down' state and vice versa or for the scattering between the same spin states but of different channels at the first and second junctions, respectively.

In accordance with the consideration given in section 2, there are no spin flips ($|\uparrow\rangle \rightarrow |\downarrow\rangle$) in the two channels considered, and only the following matrix elements of r are non-zero: $r_{11}, r_{13}, r_{22}, r_{24}, r_{31}, r_{33}, r_{42}, r_{44}$ (the same is true for the matrix elements of r').

Hence, the matrices r, r' are of the form

$$r = \begin{pmatrix} r_{11} & 0 & r_{13} & 0 \\ 0 & r_{22} & 0 & r_{24} \\ r_{31} & 0 & r_{33} & 0 \\ 0 & r_{42} & 0 & r_{44} \end{pmatrix} \qquad r' = \begin{pmatrix} r'_{11} & 0 & r'_{13} & 0 \\ 0 & r'_{22} & 0 & r'_{24} \\ r'_{31} & 0 & r'_{33} & 0 \\ 0 & r'_{42} & 0 & r'_{44} \end{pmatrix}.$$

In order to construct P and P', it is necessary to note that the spin parts of the wave functions acquire phase factors due to Larmor spin precession about the $\boldsymbol{B}$-axis. Since the magnetic fields in the channels are different, these phase factors are also different.

If one treats the 'spin-up' and 'spin-down' states as two opposite points on a unit sphere S^2, which can be transformed into one another under rotation by an angle $\varphi = \pm\pi$ about some axis $\boldsymbol{a}$, then the matrix elements describing the phase shifts in the two channels can be written as

$$P_{\pm 1} = \exp(\pm \mathrm{i}\varphi_{\boldsymbol{a}}) \exp(\mathrm{i}\theta_{1,\boldsymbol{b}}) \qquad P'_{\pm 1} = \exp(\pm \mathrm{i}\varphi_{\boldsymbol{a}}) \exp(-\mathrm{i}\theta_{1,\boldsymbol{b}}) \tag{8}$$

$$P_{\pm 2} = \exp(\pm \mathrm{i}\varphi_{\boldsymbol{a}}) \exp(\mathrm{i}\theta_{2,\boldsymbol{b}}) \qquad P'_{\pm 2} = \exp(\pm \mathrm{i}\varphi_{\boldsymbol{a}}) \exp(-\mathrm{i}\theta_{2,\boldsymbol{b}}). \tag{9}$$

Here we have also formally introduced the $\boldsymbol{b}$-axis (the subscript of $\theta_{1,2}$) which is a unit vector along the precession axis; $+\boldsymbol{b}$ corresponds to the electron propagation from $x = 0$ to $x = L$ while $-\boldsymbol{b}$ corresponds to reverse propagation; θ_1 and θ_2 are the phases acquired by the spin parts of the wave functions in the channels 1 and 2, respectively.

The idea of (8) and (9) is to express the elements of the matrices P, P' as two rotations about two independent axes. Then, these objects are nothing but the unitary quaternions [12]. As is known [13], quaternions make a real four-dimensional vector space and, since the two channels 1 and 2 are supposed to be isolated, the matrices P and P' are diagonal 4×4 matrices with the diagonal elements defined by (8) and (9).

After a great deal of algebra (see appendix 2), we have

$$|T|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 + |a_4|^2 + (a_1^* a_3 + a_1 a_3^* + a_2^* a_4 + a_2 a_4^*) + (a_1^* a_2 + a_1 a_2^* + a_2^* a_3 + a_2 a_3^* + a_3^* a_4 + a_3 a_4^*) \cos \Delta\theta \qquad \Delta\theta = \theta_1 - \theta_2$$

where a_i $(i = 1, 2, 3, 4)$ do not depend on θ_1, θ_2 and are complicated functions of $r_{ij}, r'_{ij}, t_i, t'_i$.

So, the problem now is that of how to calculate the additional phase shift $\Delta\theta = \theta_1 - \theta_2$ which arises due to the precession of the electron spin in the periodic magnetic field of the micromagnets.

4. Calculation of the phase shift

Consider the non-relativistic motion of the particle (electron) with the spin $s = 1/2$ in a two-component magnetic field: $\boldsymbol{B} = \boldsymbol{B}_0 + \boldsymbol{B}_1$, $\boldsymbol{B}_0 = (0, B_0, 0)$ and $\boldsymbol{B}_1 = (0, 0, B_1(x))$, where $B_1(x)$ is given by (1). The spin part of the electron wave function can be considered as a two-component vector defined by the pair of functions $\chi(|\uparrow\rangle)$ and $\chi(|\downarrow\rangle)$ which stand for the probability amplitudes of the two possible orientations of the spin. The spin operator $\hat{\sigma}(\sigma_x, \sigma_y, \sigma_z)$ is defined in terms of the Pauli matrices:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma_y = \begin{pmatrix} 0 & -\mathrm{i} \\ \mathrm{i} & 0 \end{pmatrix} \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Thus, we can treat the mean value of the magnetic moment of the electron moving within the channels of the microstructure as the classical quantity $\boldsymbol{P} = \langle\sigma\rangle$, its evolution under a magnetic field being defined by the equation

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} = \gamma\,[\boldsymbol{P}, \boldsymbol{B}]$$

where $\gamma = e/mc$ is the electron gyromagnetic constant.

In other words, the vector $\boldsymbol{P}$ can be treated as a classical magnetic top and, if this classical top having the initial orientation $\boldsymbol{P}_0 = (P_x^0, P_y^0, P_z^0)$ enters the magnetic field $\boldsymbol{B} = (B_x, B_y, B_z)$, it begins to precess about the magnetic field with the frequency $\Omega = \gamma B$, where $B = \sqrt{\{B_x^2 + B_y^2 + B_z^2\}}$. In the reference frame of the electron moving with the velocity v in the space domain occupied by the magnetic fields $\boldsymbol{B}_0$, $\boldsymbol{B}_1$, the varying field of the frequency $\omega(v) = 2\pi v/(a+b)$, where $a+b$ is the half-period of the magnetic grating, affects the electron. This oscillating field in the reference frame rotating with the frequency $\omega(v)$ is of the form

$$B_x = 0 \qquad B_y = B_0 - \omega(v)/\gamma \qquad B_z = B_1(x).$$

The components of the field $\boldsymbol{B}(v, x)$ can be expressed in terms of the angle $\phi(v, x)$ between the field $\boldsymbol{B}$ and the x-axis:

$$B_y = B(v, x)\sin\phi(v, x)$$
$$B_z = B(v, x)\cos\phi(v, x)$$
$$\phi(v, x) = \arcsin(B_y(v, x)/B(v, x)).$$

Let us introduce now the phase of the precessing spin by means of the formula

$$\theta(v, x) = (\mu_B/\hbar)\int_0^x B(v, x)\,\mathrm{d}t = \gamma\int_0^x B(v, x)\,\mathrm{d}t$$

and take into account the fact that the fields $\boldsymbol{B}_0$, $\boldsymbol{B}_1$ are piecewise uniform. Then the phase of the precessing spin depends linearly on t: $\theta(v, t) = \gamma B(v)t$. Now the calculation of the phase shift $\Delta\theta$ can easily be carried out. Moreover, it is clear that under certain conditions, including that of an appropriate structure length $L = m(a + b)$, $m = 2, \ldots, N$, electron velocity and values of the magnetic fields B_0, B_1, the phase shift $\Delta\theta = \theta_2 - \theta_1$ can be a multiple of $\pi/2$. Indeed,

$$\Delta\theta = \theta_2 - \theta_1 = (n + 1/2)\pi = \gamma\sqrt{B_y^2 + B_z^2}\,L/v \qquad n = 0, 1, 2\ldots$$

where

$$B_y = B_0 - \omega(v)/\gamma = B_0 - 2\pi v/\gamma(a+b) \qquad B_z = B_1.$$

If the values of B_1, L, v, n are given, the value of B_0 which is needed for $\Delta\theta$ to be equal to a multiple of $\pi/2$ can be easily calculated:

$$B_0 = \frac{2\pi v}{\gamma(a+b)} \pm \left[B_1^2 + \frac{4\pi^2 v^2}{\gamma^2 (a+b)^2} \frac{2m^2 - n^2 - 1}{m^2} \right]^{1/2}.$$

Hence, changing the external magnetic field B_0, one can change the phase shift and the quantum interference from constructive to destructive and back. Also, it can be seen that $\Delta\theta = \theta_2 - \theta_1 = f(B_0, B_1, v)$ is a function of B_0, B_1, v. That is, the phase shift is generally speaking different for electrons with different velocities. At first sight, this makes matters worse, because it means that the 'interference pattern' should be blurred. One should remember, however, that the temperature is considered to be sufficiently low. That is, the electron distribution function $f(E) = \chi(E_F - E)$ and $v = v_F$, where $\chi(\cdots)$ is the Heaviside step-like function and E_F, v_F are the Fermi energy and Fermi velocity, respectively. So, the calculation by means of (3) can now be easily carried out and we have

$$I = (2e/h)K(A + D\cos\Delta\theta(v_F))$$

where K, A, D are coefficients depending on the peculiarities of the structure. Now it is clear that on changing B_0 one can approach very strong modulation of the conductance and, since $A \sim D$, the 'contrast' of the 'interference pattern' is defined only by the ratio

$$\sqrt{\frac{E_F - k_B T}{E_F}}$$

which at a temperature of about 40 K is of the order of 90%.

5. Conclusions

A simple theory of the quantum interference due to Larmor precession of an electron spin in a loop semiconductor mesoscopic structure is presented in this paper. Also, we assumed here 'ballistic spin transport'—that is, the phase-relaxation length $L_\varphi^{(s)}$ of the spin part of the electron wave function is assumed to be greater than the microstructure length L. If in one of the arms of the microstructure there is an additional periodic magnetic field, the spin part of the wave function acquires a phase shift due to additional spin precession about that field. If in addition we suppose the microstructure length to be chosen to be greater than $L_\varphi^{(e)}$, it is possible to 'wash out' the quantum interference related to the phase coherence of the 'orbital' part of the wave function, retaining at the same time that related to the phase coherence of the spin part and, hence, reveal the corresponding conductance oscillations. By introducing a quaternion representation of the elements of the propagation matrices, one can calculate the transmission coefficients of the structure. It is shown that strong modulation of the conductance could be achieved.

In conclusion, it is worth emphasizing that, in spite of the apparent similarity of this effect and the Aharonov–Bohm one, there is a significant difference between them. The situation is quite similar to the case of the 'Aharonov–Bohm effect' with neutrons, which has been discussed recently by Peshkin [14] and which is sometimes called the 'scalar Aharonov–Bohm effect' (the SAB effect). Peshkin compared the SAB effect with the electrical version of the usual AB effect (the so-called EAB effect) and argued that, in spite of the apparent similarity, the SAB effect has little to do with the AB or EAB effect, because, unlike the latter,

the former is brought about by an ordinary action of the Maxwell field and, hence, has all the properties of all of the other local interactions and shares none of the topological features of the AB or EAB effect. The AB effect is non-local, in that the electron experiences no force and exchanges no momentum, energy or angular momentum with the electromagnetic field.

In our case the Hamiltonian and the equation of motion also involve a contemporaneous Maxwell field in the domain of the electron's position; the effect is not topological in character and that is why we used the term 'quantum interference due to Larmor precession of the electron spin' for its characterization.

Appendix 1

Here we derive formula (5) for the spin-relaxation time, which is identified with the spin-coherence destruction due to the scattering by lattice vibrations. Recall that the model accepted above for the phonon bath is this: we have a great number ($N \gg 1$) of identical non-interacting subsystems $\mathcal{B}_n$ with excitation energy ε; the Hamiltonian of the whole system—that is, the subsystem $\mathcal{A}$ together with phonon bath—is given by (3) while the interaction Hamiltonian $\mathcal{H}_{int}$ is given by (4). Since the interaction Hamiltonian (4) commutes with the operator (3), the statistical operator ρ of the whole system obeys the equation

$$\mathrm{i}\hbar\frac{\partial\rho(t)}{\partial t} = [\mathcal{H}_{int}, \rho(t)]$$

with the initial condition

$$\rho(0) = \rho_a \prod_n^N \rho_n.$$

Using the explicit formula (4), we obtain the following system of finite-difference equations:

$$\rho(n\tau + \tau) - \rho(n\tau) = \frac{\tau}{\mathrm{i}\hbar}\,[H_{n+1}, \rho(n\tau + \tau]\,.$$

From this system, by means of successive approximations one has

$$\rho(n\tau + \tau) - \rho(n\tau) = \frac{\tau}{\mathrm{i}\hbar}\,[H_{n+1}, \rho(n\tau)] + \frac{1}{2}\left(\frac{\tau}{\mathrm{i}\hbar}\right)^2 [\mathcal{H}_{int}, [\mathcal{H}_{int}, \rho(n\tau)]] + \cdots.$$

Introducing $\rho_a = \mathrm{Sp}_T\,\rho$, the statistical operator of the subsystem $\mathcal{A}$, where $\mathrm{Sp}_T(\cdots)$ is the trace operator, and supposing that

$$\frac{\partial\rho(t)}{\partial t} = [\rho_a(n\tau + \tau) - \rho_a(n\tau)]\,\tau^{-1}$$

we obtain the following kinetic equation:

$$\frac{\partial\rho_a(t)}{\partial t} = -\frac{\omega_i}{2}\{\langle b_n b_n^\dagger\rangle([a^\dagger a, \rho_a(t)] - 2a\rho_a a^\dagger) + \langle b_n b_n^\dagger\rangle([aa^\dagger, \rho_a(t)] - 2a^\dagger\rho_a(t)a)\}. \quad \text{(A.1)}$$

Here, the parameter $\omega_i = \tau\varepsilon_{int}^2/\hbar^2$ has the dimension of frequency and $[A, B] = AB - BA$. Let us note that similar equations were considered earlier in [15, 16].

In the occupation number representation, the operators $a, a^\dagger, \rho_a(t)$ are defined on the eigenfunction space of the occupation number operator $aa^\dagger$; its eigenfunctions $|n\rangle$ are (recall that the subsystem $\mathcal{A}$ is a two-level system)

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Then,

$$a^{\dagger}a = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \qquad a = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad a^{\dagger} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

Let us now suppose that the solution of (A.1) is of the form

$$\rho_a(t) = \sum_{n=1}^{2} p_n(t)\sigma(n) \tag{A.2}$$

where the matrices $\sigma(n)$ are $\sigma(1) = a^{\dagger}a$, $\sigma(2) = aa^{\dagger}$; they obey the relation

$$\mathrm{Sp}\{\sigma(n)\sigma(n')\} = \delta_{nn'}.$$

By means of these expressions, we obtain $p_n(t) = \mathrm{Sp}_a\{\rho_a(t)\sigma(n)\}$ and, hence, $p_1(t)$ and $p_2(t)$ are the probabilities that the subsystem $\mathcal{A}$ is in an excited state and an unexcited one, respectively. By means of (A.1) and (A.2), we obtain

$$\sum_{n=1}^{2} \sigma(n)\frac{\partial p_n}{\partial t} = -\omega_i \langle b_n^{\dagger} b_n\rangle(\sigma(2) - \sigma(1))p_1 - \omega_i \langle b_n b_n^{\dagger}\rangle(\sigma(1) - \sigma(2))p_2.$$

Using again the relation $\mathrm{Sp}\{\sigma(n)\sigma(n')\} = \delta_{nn'}$, one can transform this expression into the following system of equations:

$$\frac{\partial p_1(t)}{\partial t} = \omega_i(\langle b_n^{\dagger} b_n\rangle p_2(t) - \langle b_n b_n^{\dagger}\rangle p_1(t))$$
$$\frac{\partial p_2(t)}{\partial t} = \omega_i(\langle b_n b_n^{\dagger}\rangle p_1(t) - \langle b_n b_n^{\dagger}\rangle p_2).$$

From these two, it follows that $p_1(t) + p_2(t) =$ constant, and

$$\frac{\partial p_1(t)}{\partial t} = \omega_i(\langle b_n^{\dagger} b_n\rangle - Bp_1(t))$$

where $B \equiv \langle b_n^{\dagger} b_n\rangle + \langle b_n b_n^{\dagger}\rangle$. The solution of the above equation with the initial condition $p_1(t) = p_0 \leqslant 1$ is of the form

$$p_1(t) = \frac{\langle b_n^{\dagger} b_n\rangle}{B} + \left(p_0 - \frac{\langle b_n^{\dagger} b_n\rangle}{B}\right)\exp(-\omega_i Bt).$$

We are interested in estimating the time which is needed for the subsystem $\mathcal{A}$ to achieve the state with $p_1 = p_2 = 1/2$. Hence, from the above equation we have

$$1/2 - \frac{\langle b_n^{\dagger} b_n\rangle}{B} = \left(1 - \frac{\langle b_n^{\dagger} b_n\rangle}{B}\right)\exp(-\omega_i Bt). \tag{A.3}$$

Let us recall that we suppose the subsystems $\mathcal{B}_n$ to have equidistant spectra; then, it is well known that

$$\langle b_n^{\dagger} b_n\rangle = \mathrm{Sp}(\rho_n b_n^{\dagger} b_n) = (\exp\beta\varepsilon - 1)^{-1}$$
$$\langle B\rangle \equiv \langle b_n^{\dagger} b_n\rangle + \langle b_n b_n^{\dagger}\rangle = \coth(\beta\varepsilon/2).$$

Now, it immediately follows from (A.3) that

$$t = \omega_i^{-1}\tanh(\beta\varepsilon/2)\left[\ln 2 + \ln\left|\frac{\exp(\beta\varepsilon) - \tanh(\beta\varepsilon/2) - 1}{\exp(\beta\varepsilon) - 2\tanh(\beta\varepsilon/2) - 1}\right|\right] \approx \omega_i^{-1}\ln 2\tanh(\beta\epsilon/2).$$

This is formula (5).

Appendix 2

Here we outline briefly how the formula for $|T|^2$ can be obtained. Let us start from formula (9), where the matrix t is a 4×1 matrix describing the transmission from the left-hand end of the structure into the channels while t' is a 1×4 matrix describing the transmission from the channels into the right-hand end:

$$t = \begin{pmatrix} t_{+1} \\ t_{-1} \\ t_{+2} \\ t_{-2} \end{pmatrix} \qquad t = (\, t'_{+1} \quad t'_{-1} \quad t'_{+2} \quad t'_{-2}\,).$$

Here the subscripts ± 1, ± 2 stand for the channels 1, 2 and the states $|\uparrow\rangle(+)$ and $|\downarrow\rangle(-)$. Multiplying the matrices P, r, P', r', one can obtain for $[I - PrP'r'] = M$ the following expression:

$$M = \begin{pmatrix} 1 - A_{11} & 0 & A_{13} & 0 \\ 0 & 1 - A_{22} & 0 & A_{24} \\ A_{31} & 0 & 1 - A_{33} & 0 \\ 0 & A_{42} & 0 & 1 - A_{44} \end{pmatrix}$$

where

$$\begin{aligned} A_{11} &= P_{+1}^2 r_{11} r'_{11} + P_{+1} P_{+2} r_{13} r'_{31} & A_{13} &= P_{+1}^2 r_{11} r'_{13} + P_{+1} P_{+2} r_{13} r'_{33} \\ A_{22} &= P_{-1}^2 r_{22} r'_{22} + P_{-1} P_{-2} r_{24} r'_{42} & A_{24} &= P_{-1}^2 r_{22} r'_{24} + P_{-1} P_{-2} r_{24} r'_{44} \\ A_{31} &= P_{+2} P_{+1} r_{31} r'_{11} + P_{+2}^2 r_{33} r'_{31} & A_{33} &= P_{+2} P_{+1} r_{31} r'_{13} + P_{+2}^2 r_{33} r'_{33} \\ A_{42} &= P_{-2} P_{-1} r_{42} r'_{22} + P_{-2}^2 r_{44} r'_{42} & A_{44} &= P_{-2} P_{-1} r_{42} r'_{24} + P_{-2}^2 r_{44} r'_{44}. \end{aligned}$$

With matrix M inverted, one gets

$$M^{-1} = \begin{pmatrix} 1 - A_{11} & 0 & A_{13} & 0 \\ 0 & 1 - A_{22} & 0 & A_{24} \\ A_{31} & 0 & 1 - A_{33} & 0 \\ 0 & A_{42} & 0 & 1 - A_{44} \end{pmatrix}^{-1} = \begin{pmatrix} \alpha_{11} & 0 & \alpha_{13} & 0 \\ 0 & \alpha_{22} & 0 & \alpha_{24} \\ \alpha_{31} & 0 & \alpha_{33} & 0 \\ 0 & \alpha_{42} & 0 & \alpha_{44} \end{pmatrix}$$

where

$$\begin{aligned} \alpha_{11} &= \Delta^{-1}(1 - A_{33}[(1 - A_{22})(1 - A_{44}) - A_{24}A_{42}] \\ \alpha_{13} &= -(A_{31}/\Delta)[(1 - A_{22})(1 - A_{44}) - A_{24}A_{42}] \\ \alpha_{22} &= \Delta^{-1}(1 - A_{44})[(1 - A_{11})(1 - A_{33}) - A_{31}A_{13}] \\ \alpha_{24} &= -(A_{13}/\Delta)[(1 - A_{11})(1 - A_{33}) - A_{13}A_{31}] \\ \alpha_{31} &= -(A_{13}/\Delta)[(1 - A_{22})(1 - A_{44}) - A_{24}A_{42}] \\ \alpha_{33} &= \Delta^{-1}(1 - A_{11})[(1 - A_{22})(1 - A_{44}) - A_{24}A_{42}] \\ \alpha_{42} &= -(A_{24}/\Delta)[(1 - A_{11})(1 - A_{33}) - A_{31}A_{13}] \\ \alpha_{44} &= \Delta^{-1}(1 - A_{22})[(1 - A_{11})(1 - A_{33}) - A_{13}A_{31}] \end{aligned}$$

$$\Delta = \prod_{i=1}^{4}(1 - A_{ii}) - (1 - A_{11})(1 - A_{33})A_{24}A_{42} - A_{13}A_{31}[(1 - A_{22})(1 - A_{44}) - A_{24}A_{42}].$$

Inserting these formulae into (7), after another round of tiresome multiplication of matrices, one gets the formula for the squared transmission coefficient $|T|^2$.

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